The Laboratory Write-up

Table of Contents

The Anatomy of a Lab Report

Everyone knows that laboratory experiments are intended to reinforce your understanding of basic concepts by providing you with some hands on experience. But that is only one purpose of the laboratory. Another, of equal importance, is for you to learn how to perform an investigation and present the results of that investigation in a manner that is informative, interesting and to the point. Just like a professional scientist, you must be able to demonstrate your understanding of the concepts involved in the experiment as well as the objectives of the experiment. You must also be able to provide a relevant interpretation of the results of your experiment.

If this sounds vague, as it should, allow me to provide a practical interpretation of what is expected in your write up, namely the "what," "why," "how" and "what" of your experiment.

1. What you are doing
2. Why you are doing it (what you hope to confirm or find out)
3. How you are doing it
4. What your results are (and what do they tell you.)

There are many different methods of presenting the necessary information about an experiment. As you become more familiar with experimental work, you will develop your own style. For now, however, you should do your write-ups according the following guidelines, modeled after articles appearing in professional journals such as American Journal of Physics or American Journal of Astronomy and Astrophysics.

There is one important distinction between your write-up and professional journal articles, however, That distinction is one of formality. Journal articles are often written in "journalese," in which the authors rarely, if ever speak of themselves directly, and often use the passive voice in describing their investigation. In undergraduate physics, you should strive for a balance between the objective tone of a researcher, and the active voice of a journalist reporting on a event for the benefit of your classmates and your instructor.

Your write-up should be divided into three or more sections, depending on the type of experiment. Dividing up the report makes it more understandable to the reader. The remainder of this section provides guidelines for what these sections should contain.

Introduction

The first section is called the “Introduction". This section is intended to guide the reader through your experiment. In this section, you introduce the concepts involved in the experiment and then "talk the reader through" the details of your investigation. Imagine that the introduction is for the benefit of colleagues who are looking through many write-ups such as your own. They are looking for specific results that may verify, or nullify their own results. Of course, numbers alone will have no meaning to them. They must know if the experiment is of interest to them and if the methods are valid from their point of view.

In other words, they want to know points 1- 4, listed above (what, why, how, results). If the introduction is done properly, your colleague (or your instructor) will know all he needs to know about your investigation, unless, of course, he is curious about the specific details of your results. In that case he will read on.

Data

If your introduction is property written, then the reader will know what measurements were made in the experiment. He or she, however, will not understand the meaning of various numbers scribbled on the paper unless they are organized in tables. Tables, and the values contained therein, should be discussed in the intro.

Tables should be titled, and columns and rows should be clearly labeled by a word and a symbol. It is often helpful to include raw data and the results of calculations in the same table. You are free to use as many separate tables as you choose. Sometimes, tables (excluding data, of course) will be provided for you. Otherwise, you must construct your own data tables. An example follows:

Height *h*	Uncertainty (δh)	t_{theory} $t=(2h/g)^{.5}$	t_{exp}	Difference as $\lvert t_{exp} - t_{theory} \rvert$	Uncertainty t_{theory}	Uncertainty t_{exp}
1.0 m	.01 m	.45 s	.5 s	.05 s	.05 s	.002 s
2.0 m	.01 m	.64 s	.7 s	.06 s	.06 s	.003 s

Table 1. Free Fall Data

Note that this table is titled "Free Fall Data," and has each measurement and calculation clearly labeled. Equally important are the units (ie: s, m, lbs, etc) attached to each number. It is (almost) possible to understand what went on in the experiment by inspecting the data table. You do not always have to write out the formula at the top of each column, as I did in column three, although do it if you think it will help the reader to understand what you are doing. Do include the symbol for the measured or theoretical quantity (i.e., t_{exp} in column 4). In this example, as with the majority of experiments you will perform, the goal is to verify a formula. In this case, the formula predicts the time it will take an object to fall from a given height. There are only two measurements: the height, and the time for the object to fall. Both measurements are included in the table under "h" and "t_{exp}" respectively. "t_{theory}" is the prediction of the formula you are attempting to-verify, and this value depends on the value of h.

Data Analysis

This is the part of the write up where you really show your stuff -- where you demonstrate, through calculations, graphs, charts, etc., what conclusions can be drawn from the experiment. Think of this part of the write-up as being like a tax audit -- you need to prove to the IRS (or your instructor) why you are able to make a particular claim (of course, if you don't make any claims, then you don't have anything to prove, but then you don't get the benefit of nice refund check, or at least a good grade). The Data Analysis section may include the following:

Sample Calculations

Most experiments will involve calculations of an expected or theoretical value, based on measurements you make in the laboratory. Calculations of uncertainties are also necessary in most laboratory experiments. You must demonstrate at least one sample calculation of each type, even if you can do it on your calculator. This is not for purely sadistic reasons. In order for the reader to understand how you arrived at your results, he must know exactly what you did. He cannot attempt to duplicate your work if he is not aware of what you are doing each step of the way.

Graphs and Charts

These are often required in the laboratory assignment (ie: plot the time of fall as a function of height). Even if they are not required, many students find it helpful to graphically depict the results of their analysis. If graphs are included in your write-up, do them neatly, using a computer program if possible, or at least a straight edge and millimeter graph paper, Graphs should be titled, with axes labeled neatly. As with tables, graphs should be discussed in the Abstract and/or the summary.

Uncertainty Analysis

Note that three uncertainties are also listed in the table above. Since we are measuring the height with a meter stick, there will be some uncertainty in how well we can read its markings. To be safe, I used 1 cm (.01 meters) as the uncertainty in this reading.

Since we are not certain about the value of h, we cannot be certain about the value of t_{theory} which depends on h. We will learn more about how uncertainties propagate through calculations later. For now it only important to realize that they do,

There is also an uncertainty in t_{exp}, the time you measure with a stopwatch for the object to fall. How do you determine this uncertainty? The easiest, most reliable method is simply to repeat the experiment a number of times. In all likelihood, each measurement will be slightly different from the others and from the average. The uncertainty will then be the average of these "deviations from the average" (otherwise known as the absolute average deviation). More on this later...

Conclusion/Summary

It may be that your results are not easily understood (for example, they do not agree with the theoretical predictions) and cannot be explained briefly in the abstract. In that case, you will have a separate section at the end of the report entitled: "Summary," or "Conclusions," or both, depending on the experiment. What you write in this section is entirely up to you. Usually, this section describes:

- What you think went wrong with the experiment (why your data did not agree with the theoretical values).
- What you could or would do if given the chance to improve the agreement between experiment and theory.
- What modifications should be made in the theory (if any) to account for the discrepancy between experiment and theory; i.e., air resistance was not taken into account, or perhaps the amplitude of the pendulum exceeded the small amplitude approximation made in computing the theoretical value.

Just making a guess about what factors were not considered is not usually enough! You must explore the consequences of your hypothesis. For example, consider an experiment to measure the force applied to a cart. From Newton's Second Law, *Force = Mass x acceleration* ($F = ma$). Pretend that you have applied a known force to a small car on a tabletop and measured the mass and the acceleration of the car. What if the product of mass times acceleration is greater than the known force. What went wrong? For example:

- **Hypothesis #1: Friction in the wheels.** Friction in the wheels would prevent the wheels from spinning easily, thus acting like a counter force, and thus produce a smaller acceleration, so the measured force would be less than the known applied force. Clearly, this could not be the problem.
- **Hypothesis #2: The Table was actually slanted downward.** If the table was slanted, then gravity would pull the cart downward, providing a helping force, and thus increasing the acceleration, The experimentally determined force would then be larger then the known force, in accordance with the experimental results.

If you can make some estimate of the numbers involved, and their effects on your final result, then your conjecture will seem far more accurate. For example, consider the following passage:

> "To account for the 1.5% difference between the known force and the experimental force, the table would only have to be tilted by 3 degrees. An incline this shallow could easily go undetected without the use of a level."

Interpreting your Results

A common mistake made by lower division students is reporting that "the experiment was a success, " if the hoped-for result was achieved. In physics lab, however, the terms “success" and "failure" will be replaced by the more exciting (or at least meaningful) terms: "verified within the limits of uncertainty," and "not verified within the limits of uncertainty," respectively. For example, let's say you wish to verify a formula for free fall, which states that the time it takes for a object to fall a distance, *h*, is given by the formula:

$$t = \sqrt{\frac{2h}{g}}, \text{ where } g = 9.8\frac{m}{s^2}$$

Let's say you measure the height and find it to be 1.00 meters. The formula predicts that the time for the ball to fall to the ground ought to be 0.45 seconds. Now you drop the ball and it takes 0.4 seconds one time, 0.6 seconds the next time, and 0.5 seconds the time after that. While there are many "acceptable" ways to determine the uncertainty in your values, the most common and straightforward method involving a small number of trials is to use the maximum experimental value (0.6), the minimum value (0.4) and take "half the range" $(0.6 - 0.4)/2 = 0.05$ seconds. In addition, there is an uncertainty in the theoretical value t_{theory} of 0.002 seconds (take my word for it), giving a total uncertainty in the free fall time of 0.052 seconds. Now, what can you say about the results of the experiment?

Before we try to answer that question, note that the experimental time and the theoretical time are not the same. In fact, for $h = 1.00$ meters, they differ by 0.05 seconds. However, the total uncertainty is greater than that difference (0.052 seconds). So even though the values you have for the experimental value and the theoretical value do not agree numerically, they do agree to within your ability to judge. What you might say is: "The average time for the ball to fall 1.00 meters was 0.50 ± 0.052 seconds. Since the free fall formula predicts a free fall time of 0.45 seconds, we found the predictions of this formula to be valid within the limits of our experiment."

It may not sound exciting, but it is honest. You should accurately, and honestly report what happened in the lab and how it compared to what you expected. Often, you will learn more when the theoretical results do not lie within your experimental uncertainty. If you can explain why, then your experiment is also done successfully.

Error Analysis

Contrary to what you might have heard or experienced in previous courses, no methods of data analysis existed before Homo sapiens began using his large brain to make sense out of the world. Concepts like "standard deviation," "mean," and even “uncertainty," weren't carved in stone somewhere. They were invented to allow scientific types (however prehistoric) to communicate with one another.

Think about it: What could you say about a page (or pages) of data other than. "here, look ... see?" Here are the kinds of things we say about data in the physics laboratory:

Uncertainty

When you make a measurement such as distance, temperature, time, etc., your measurement is only as accurate as your measuring instrument. In physics, these uncertainties are of special interest since the predictions of competing theories often differ minutely from one another. It is important for you to learn to incorporate uncertainties into your results ... Nothing is for certain!

A single measurement, such as the time it takes a ball to fall from a given height, will have an uncertainty, δt, that depends on the measuring instrument. The best system available in the undergraduate laboratory is a photogate timer that judges how long an optical beam is interrupted. Let us imagine that the timer's digital display reads: 0.016. What is the uncertainty?

Answer: at least 0.001 seconds. With a digital timer, there is no way to know whether the last digit is about to change. We are always uncertain about the last digit. However, The timer itself has a 0.2% tolerance. This means that there is an additional uncertainty of *(0.002)(0.016) seconds = 0.000032* seconds. In this case, the tolerance results in a vanishingly small uncertainty. Of course, this may not always be the case.

If you (or your instructor) are not satisfied with an uncertainty as large as may occur in a single measurement, what can be done?

Answer: Repeat the measurement as many times as possible. Then your uncertainty in the value of a particular quantity can be determined from the spread or range in values that you obtain during the experiment. Repeating the measurement a number of times causes the individual measurement uncertainties to cancel more or less completely -- especially when the measurement uncertainties are much smaller than the range of values obtained in the experiment, and if you make enough measurements to achieve a random distribution of values. Doing this, however, opens up a can of worms called statistics, so let's dive on in!

Statistics for Physics: A Short course

In this section you will find some basic statistical equations you will need for performing error and uncertainty analysis on your experimental results. Keep them handy!

mean (<x>)

Imagine you do something (like fire a projectile) a number of times, and measure the distance *W* that the projectile travels each time. If you make *N* measurements then the mean value of *x* is given by

$$< x > = \frac{\sum_{i=1}^{N} x_i}{N}$$ **Eq. 1**

Example:

Given the following *x* data in meters (m):

X_1	= 10	X_2	= 15	X_3	= 8.75
X_4	= 16.25	X_5	= 20	X_6	= 5

$$< x > = \frac{10.0 + 15.0 + 8.75 + 16.25 + 20.0 + 5.0}{6} = 12.5$$

Absolute average deviation of the mean (δx)

What is the average difference between the mean value (3.02m) and a typical experimental value? The answer is the absolute average deviation, or the average deviation for short.

$$\delta x = \frac{\sum_{i=1}^{N} \left| x_i - < x > \right|}{N}$$ **Eq. 2**

Note that the each term is positive regardless of weather the mean is the larger or smaller number.

Example:

Using the values for *x* from the first example,

$$\delta x = \frac{|10 - 12.5| + |15 - 12.5| + |8.75 - 12.5| + |16.25 - 12.5| + |12.0 - 12.5| + |15.0 - 12.5|}{6} = 4.5m$$

The "half range" approximation

Note that when the number of values is small (less than five) a handy approximation for the average deviation is simply to take half the range.

$$\delta x = \frac{x_{\max} - x_{\min}}{2}$$ **Eq. 3**

Example:

Using the data from above, we find:

$$\delta x = \frac{20 - 5}{2} = 7.5m$$

In this case, the half-range value is much larger then the average deviation -- due to the fact that at least one value (the first one) is extremely large. With such a small amount of data, this large value could be an exception, thus illustrating the reason for taking a large number of measurements. On the other hand, if you only have a few data values, and these values are closely spaced, then the half range method is the one to use. Note that if you have three values; 1,2,3 for example; the mean value is 2 and the absolute average deviation is [(2-1) + (2-2) + (3-2)]/3 = [1+0+1]/3 = 2/3 or .67. Using the half range method, your uncertainty would be: [3 - 1]/2 = 1. Clearly, 1 is a better measure of your experimental uncertainty than .67 in this case. On the other hand, if you have the follwing numbers; x = 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 6; the half the range would be [6-1]/2 = 2.5 which is much too large! In this case, the absolute average deviation would be a much better method of determining the uncertainty.

Standard deviation from the mean (σ_x)

The standard deviation is another measure of the average difference between an experimental value and the mean value. This measure is useful when the data is symmetrically distributed about the mean value and follows a bell curve distribution like the five values for x used here.

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \langle x \rangle)^2}{N-1}} \quad \textbf{Eq. 4}$$

Example:
Using projectile data above, we find that:

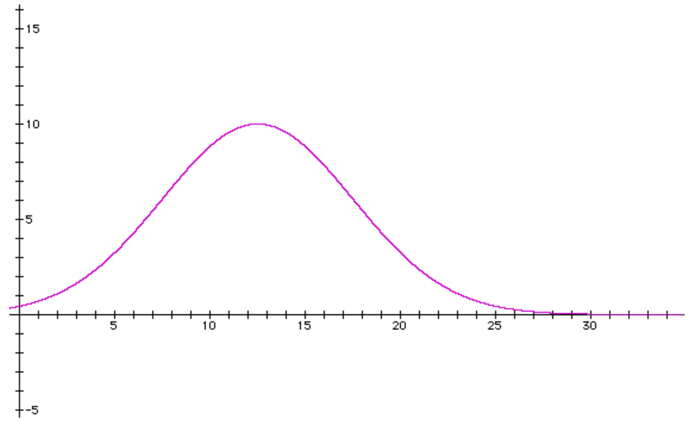
$$\sigma_x = \sqrt{\frac{2.5^2 + 2.5^2 + 3.25^2 + 3.75^2 + 7.5^2 + 7.5^2}{5}} = 5.5m$$

Note: use of the standard deviation requires a large number of values, usually far more than 5; I cheated here by choosing five values from the curve below that are symmetrically distributed about the mean value.

IMPORTANT: If the data is not randomly distributed about the mean, then the standard deviation has no physical meaning (although people use it anyway).

The Gaussian curve

Say we continue firing the projectile from the same height with the some gun one hundred times, and we measure both the distance from the gun that the projectile lands, W, as well as the number of projectiles, $N(x)$, that land a distance a distance, x, away. A plot of $N(x)$ *vs.* x is shown below.



What if your employer calls you into his office and says, "Well, sharp guy, what were the results of the projectile experiment?" What do you tell him? Before we answer, lets look at the curve. Notice how the data points are symmetrically distributed about a mean value (12.5 m) and form a bell shaped (or "Gaussian") curve. This type of distribution indicates that trying to predict exactly where a projectile will land is no easier than trying to predict what slot a roulette ball will fall into. You can assign a probability to each value of *x*, but not more.

Actually, this type of curve is described by the equation:

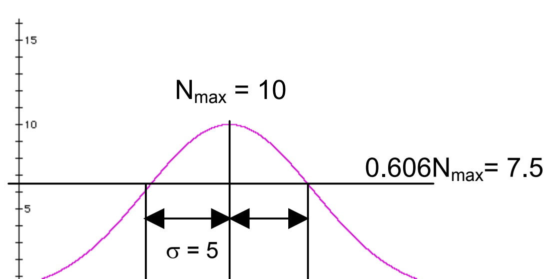
$$N(x) = \frac{N}{\sigma\sqrt{2\pi}} e^{-(\frac{[x-<x>]^2}{2\sigma^2})} \qquad \textbf{Eq. 5}$$

Note that $\frac{N}{\sigma\sqrt{2\pi}}$ is also equal to N_{max}.

Where:

- N = the total number of projectiles shot
- x = the distance a projectile lands from the origin
- σ = the standard deviation from the mean (more on this below)
- $N(x)$ = the number of projectiles that lands a distance x from the origin in a range between x and $x + \delta x$ (or $x + \Delta x$)
- <x> = the mean value of x
- N_{max} = the number of projectiles that land at a distance of between $<x>$ and $<x> + \delta x$

Now look one more time at the curve redrawn below. This time the values of σ have been drawn in to indicate their graphical meaning. As you can see, the value of σ is a measure of the width of the distribution.



At one standard deviation away from the mean (here 5 m), you can see that the number of projectiles has fallen from 10 at $x = 12.5\ m$, to 6.06 (10exp(-0.5)) at a distance of $x = \pm\sigma$ away from the mean (you can see this by letting $x = \sigma$ in the Gaussian equation above so that $N(x) = N_{max}exp(-1/2)$. But what does this tell us? *Answer:* read on!

Probability and the Gaussian curve

The standard deviation does tell us something about the **probability** that a projectile will land within a certain distance of the mean. In fact, the probability that an object will land within a distance of σ (here 5 m) on either side of the mean is exactly 68.3%. The probability that an object will land within a distance of 2σ of the mean is **95.45%,** and within three standard deviations (within a distance of 3σ) is 99.73%.

Probability is determined by the area under the curve. When speaking of probablilites, we generally say that the probablities of landing at any given value should sum up to 1.0. It is easy to understand why if you think about it in terms of percentages, where you have to be 100% certain (where 1.0 * 100 = 100%) that it will land <u>*somewhere*</u>! In your studies you will find that:

$$P(f(x)) = \int_{-\infty}^{\infty} f(x)dx = 1$$

where, in this case, $f(x)$ is the equation describing the gaussian curve.

If we assign the total area under the curve a value of 1, then the area under the gaussian curve between $<x> - a < x < <x> + a$ is 0.683. Similarly, the area under the curve within two standard deviations of the mean is 0.9545.

So what do you tell your employer? Well, first you hand him the graph, and then you say, "Well the mean value of x was 12.5 meters and the standard deviation was 5.00 meters." If he or she knows statistics, then he or she will realize that this projectile gun is not very accurate.

What you are actually telling him or her is that there is a 68.3% chance that the projectile will land between 7.5 and 17.5 m away from the origin, and a 95.45% chance that a projectile will land between 2.5 and 22.5 meters away from the origin. Typically such data is reported as:

$$x = <x> \pm \ \sigma$$

or in this case:

$$x = 12.5 \pm 5 \text{ in meters}$$

Thus, your "uncertainty" in this experiment would be 5 m. A more accurate projectile gun will have a much smaller value of σ, and thus the distribution would be higher and narrower.

By the way, it is not always useful, or possible to repeat a measurement -- especially in electronics, where circuit parameters such as current and voltage are read from meters within the circuit.

Determining σ from raw data

Now that you know the significance of the standard deviation, how do you find it from a bunch of *x* and *y* values? Here are the three methods you will be required to use:

- Graph the data as shown above. Find the *x* values where $y = N(x)$ is down to 60.6% of its maximum value. This *x* value is σ_x. (note: This is exactly what was done on the second graph above.)
- Use the general Equation 4 and solve for σ.
- Enter all the *x* values into your calculator or computer program (in statistics mode) and then press "standard deviation."

Relative Uncertainty (δx / x)

The fractional uncertainty in a measurement of a value *x* is simply the uncertainty (average or standard deviation) divided by the value (mean) itself. This is a useful way of describing your uncertainty, since it puts your accuracy in perspective. If you measure the length of your thumb, with an uncertainty of 1 cm, you will have a large fractional uncertainty (on the order of 0.15). If you are measuring the distance to the Moon, however, and have an uncertainty of l cm, then your relative uncertainty is more like 10^{-11}. There are two very useful formulas for relative uncertainties

- **Percent uncertainty (%x).** This is simply the relative uncertainty expressed as a percentage

$$\%x = \frac{\delta x}{x} \times 100$$

- **Percent difference (%diff).** This is the percent difference between the experimental value, and the theoretical value.

$$\%diff = \left| \frac{x_{theory} - x_{\exp}}{x_{theory}} \times 100 \right|$$

- **Averaged percent difference (%diff).** Sometimes it is difficult to determine which value is the experimental value and which is the theoretical value. In this case, use the following formula, which normalizes an average ☺:

$$avg\%diff = \frac{\left|x_{theory} \times x_{experimental}\right|}{\left(\frac{x_{theory} + x_{experimental}}{2}\right)}$$

Propagation of Errors

Often one quantity, f, depends on other quantities x, y and z as $f(x,y,z)$, all of which are measured quantities in the experiment. Each of these independent variable can have uncertainties δx, δy and δz, respectively (or, similarly standard deviations of σ_x, σ_y and σ_z). What then is the total uncertainty δf? It is found with the following formula:

$$\delta f = \left|\frac{\partial f}{\partial x}\Delta x\right| + \left|\frac{\partial f}{\partial y}\Delta y\right| + \left|\frac{\partial f}{\partial z}\Delta z\right| ...$$

Where, in this case, $\Delta(x,y,z)$ can be either the uncertainty or the standard deviation in x, y and z, respectively, depending on the type of error data computed. This type of formula, where a partial derivative of each independent variable is added together is called a “total differential.” Note the ellipses at the end of the equation, indicating that the total differential may be extended to all n independent variables.

Generally, we want to worst case scenario – just how bad our results could possibly be. Naturally, then, each of the three terms above must be positive, so that the number δf grows when each additional uncertainty is considered.

Relative Error
If $f(x,y,z)$ is of the form $x^r y^s z^t$, then the relative error is given by the relation:

$$\frac{\delta f}{< f >} = \left|r\frac{\delta x}{< x >}\right| + \left|s\frac{\delta y}{< y >}\right| + \left|t\frac{\delta z}{< z >}\right|$$

so that

$$\delta f = f\left(\left|r\frac{\delta x}{x}\right| + \left|s\frac{\delta y}{y}\right| + \left|t\frac{\delta z}{z}\right|\right)$$

Example:

If the time, t, it takes an object to fall from a height, h, is given by the relation:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{9.8}}$$

where $h = 2.00\ m$ with an uncertainty of $\pm 0.01m$, and $g = 9.80 \pm .01\ m/s^2$, what is δt?

Answer: Here, t is a function of h and g, so t plays the role of f in the δf in the relative error equation above, while h and g play the roles of x and y, respectively. Thus, $r = 0.5$ and $s = -0.5$, giving us:

$$\delta t = <t> \left| .5 \frac{\delta h}{<h>} \right| + \left| .5 \frac{\delta g}{<g>} \right|$$

In this case the average value signs are unnecessary since the height and the acceleration of gravity would be constant throughout the experiment. Thus, we find that:

$$\delta t = \sqrt{\frac{2(2.0m)}{9.8\frac{m}{s^2}}} \left(.5\frac{.01}{2.0} + .5\frac{.01}{9.8} \right) = .0019s$$

Note: In the free fall example, this method would only be useful if you could drop the ball once from a given height. A much more accurate measurement of δt would simply be to drop the ball as many times as possible and determine the average deviation or the standard deviation in t.

Uncorrelated error propagation

The method described above is called "worst case analysis;" that is, if all the uncertainties (δx, δy, and δz) add together to produce the largest possible error. If x, y, and z, are uncorrelated (do not depend on one another), then chances are that their uncertainties will tend to cancel, reducing the total uncertainty. In other words, the worse case analysis tends to be an overestimate of the uncertainty as the number of measurements increases. In that case, the uncertainty in $f(x,y,z)$ would be given by the expression:

$$\delta f = \sqrt{\left[\left(\frac{\partial f}{\partial x}\right)\delta x\right]^2 + \left[\left(\frac{\partial f}{\partial y}\right)\delta y\right]^2 + \left[\left(\frac{\partial f}{\partial z}\right)\delta z\right]^2}$$

Note: This expression is analogous to determining the length of a vector by the taking the "vector sum" of its components.

Example:

The number of detectable intelligent civilizations in the Galaxy is given by:

$$N = SpL^{-2}T^2$$

(Don't formulas like this seem authoritative?)

Where:

N = the number of intelligent civilizations

S = the number of stars in the galaxy = $5 \times 10^{10} \pm 10^{10}$
p = the average number of habitable planets per star = 1 + .5
L = the average lifetime of a civilization = $10^4 \pm 10^3$ years
T = time of observation = 1 year ± 1 hr

Given these values, how many civilizations are likely to be detected? Before we compute the percent uncertainty, note that the uncertainty in T is very small compared to the others, and can be neglected.

First, $N = (5 \times 10^{10})(1)(10^4)^{-2}(1) = 5$

Now from the uncorrelated error equation above, we have:

$$\frac{\delta N}{N} = \sqrt{\left(\left[\frac{1}{5}\right]1\right)^2 + \left(\left[\frac{.5}{1}\right]1\right)^2 + \left(\left[\frac{1}{10}\right]2\right)^2} = .57,$$

$$\text{or } \delta N = .57(5) = 2.9$$

So, there are 5 $\pm$ 3 detectable civilizations in the galaxy right now. (Where are they?!)

Note that if we had done the same analysis using the worst case method, the uncertainty δN would have been:

$$\delta N = 1/5 + .5/1 + 1/5 = .9, \text{ or } N = 5 \pm 4.5$$

In which case we might only be able to detect ourselves!

Miscellaneous Rules to Remember

- Percent or relative uncertainties add together when the values themselves are multiplied. For instance, if $f(x,y,z) = xyz$, then:

$$\%f = \%x + \%y + \%z$$

- If values are raised to a power, then the percent or relative uncertainties are multiplied to that power. For instance, if $f(x,y,z) = x^r y^s z^t$, then:

$$\%f = r(\%x) + s(\%y) + t(\%z)$$

- If values are added together, then their uncertainties add together. For instance, if $f(x,y,z) = x + y + z$, then:

$$\delta f = \delta x + \delta y + \delta z$$

- Uncertainties must be rounded so that they are no more precise than the value itself. For instance, if <*x*> = 2.36 m, and δx = 0.0168 m, then *x* must be reported as <*x*> = 2.26 ± 0.02 m.

It is occasionally convenient to “cheat” and guess at another digit for the value of *x*; for instance, $x = 2.365 \pm .017$. This means that you can guess that *x* is between 2.382 and 2.348.